Advances in Ice Sheet Model Initialization Using the First Order Model

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Estimation of ice sheet initial state

GOAL

Find ice sheet initial state that

- matches observations (surface velocity)
- is in "equilibrium" with climate forcings (SMB)

by inverting for unknown/uncertain parameters.

Significantly reduce non physical transients without spin-up.

Bibliography

- *Arthern, Gudmundsson*, J. Glaciology, 2010
- *Price, Payne, Howat and Smith*, PNAS, 2011
- *Petra, Zhu, Stadler, Hughes, Ghattas*, J. Glaciology, 2012
- *Pollard DeConto*, TCD, 2012
- W. J. J.Van Pelt et al., The Cryosphere, 2013
- *Morlighem et al.* Geophysical Research Letters, 2013
- *Goldberg and Heimbach*, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014

Perego, Price, Stadler, **Journal of Geophysical Research**, 2014

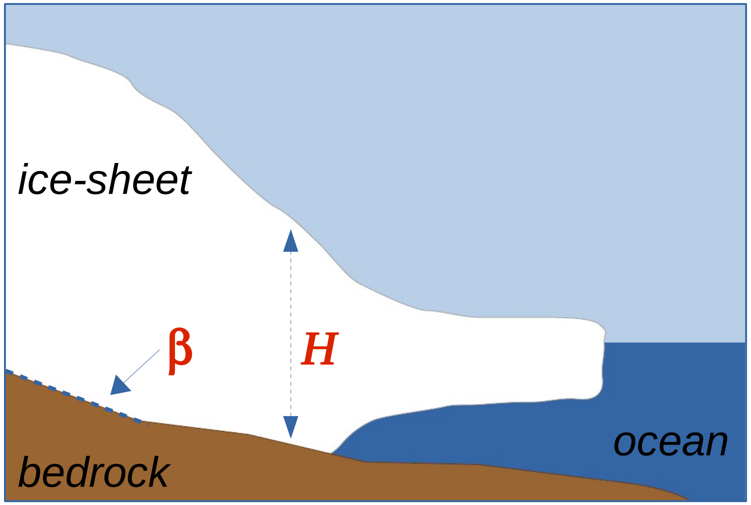
Estimation of ice sheet initial state

Problem details

Problem: **what is the initial thermo-mechanical state of the ice sheet?**

Available data/measurements

- *ice extension and surface topography*
- ***surface velocity***
- ***Surface Mass Balance (SMB)***
- ***ice thickness H (very noisy)***



Fields to be estimated

- ***ice thickness H***
- ***basal friction β***

Additional information

- *ice fulfills* ***nonlinear Stokes equation***
- *ice is almost* ***at mechanical equilibrium***

Assumption (for now)

- *given* ***temperature field***

Perego, Price, Stadler, **Journal of Geophysical Research**, 2014

Forward model: First order (FO) Stokes approximation

Model equations

$$\mathbf{D} = \begin{bmatrix} u_x & \frac{1}{2}(u_y + v_x) & \frac{1}{2}(u_z + \cancel{w_x}) \\ \frac{1}{2}(u_y + v_x) & v_y & \frac{1}{2}(v_z + \cancel{w_y}) \\ \frac{1}{2}(u_z + \cancel{w_x}) & \frac{1}{2}(v_z + \cancel{w_y}) & w_z \end{bmatrix}$$

incompressibility:

$$w_z = -(u_x + v_y)$$

quasi-hydrostatic approximation:

$$p = \rho g(s - z) - 2\mu(u_x + v_y)$$

FO is a nonlinear system of elliptic equations in the horizontal velocities:

$$\begin{cases} -\nabla \cdot (2\mu\,\mathbf{D}_1) = -\rho g \dfrac{\partial s}{\partial x} \\ -\nabla \cdot (2\mu\,\mathbf{D}_2) = -\rho g \dfrac{\partial s}{\partial y}, \end{cases}$$

$$\mathbf{D}_1 = \begin{bmatrix} 2u_x + v_y \\ \frac{1}{2}(u_y + v_x) \\ \frac{1}{2}u_z \end{bmatrix}, \quad \mathbf{D}_2 = \begin{bmatrix} \frac{1}{2}(u_y + v_x) \\ u_x + 2v_y \\ \frac{1}{2}v_z \end{bmatrix}$$

where s is the ice surface and,

$$\mu = \frac{1}{2}A^{-\frac{1}{n}}\,\dot{\varepsilon}_e^{\left(\frac{1}{n}-1\right)}, \quad \dot{\varepsilon}_e = \sqrt{\frac{1}{2}\mathrm{tr}\,(\mathbf{D}^2)}, \quad A: \text{flow factor}$$

Estimation of ice sheet initial state

Steady state equations and basal sliding conditions

How to prescribe ice sheet mechanical equilibrium:

flux divergence

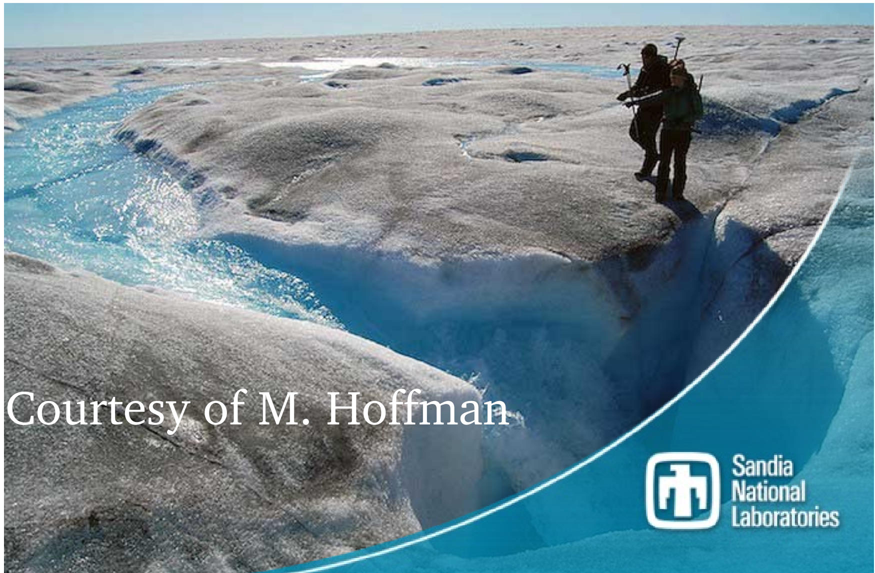
$$\frac{\partial H}{\partial t} = -\mathrm{div}\,(\mathbf{U}H) + \tau_s, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz.$$

Surface Mass Balance

$$\boxed{\mathrm{div}\,(\mathbf{U}H) - \tau_s + \left\{\frac{\partial H}{\partial t}\right\}^{obs} = 0}$$

Boundary condition at ice-bedrock interface :

$$(\sigma\mathbf{n} + \beta\mathbf{u})_{||} = \mathbf{0} \quad \text{on} \quad \Gamma_\beta$$



Courtesy of M. Hoffman

Estimation of ice sheet initial state

PDE-constrained optimization problem: cost functional

Problem: find initial conditions such that the ice is almost at thermo-mechanical equilibrium, given the geometry and the SMB, and matches available observations.

Optimization problem:

find β and H that minimizes the functional $\mathcal{J}$

$$\mathcal{J}(\beta, H) = \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds$$

surface velocity mismatch — *Common*

$$+ \int_\Sigma \frac{1}{\sigma_\tau^2} |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds$$

SMB mismatch

$$+ \int_\Sigma \frac{1}{\sigma_H^2} |H - H^{obs}|^2 \, ds$$

thickness mismatch

(surface velocity mismatch, SMB mismatch, thickness mismatch) — *Proposed*

$$+\mathcal{R}(\beta, H)$$

regularization terms.

subject to ice sheet model equations (FO or Stokes)

$\boldsymbol{U}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

Estimation of ice sheet initial state

PDE-constrained optimization problem: gradient computation

$$\text{Find } (\beta, H) \text{ that minimize} \quad \mathcal{J}(\beta, H, \mathbf{u})$$
$$\text{subject to} \quad \mathcal{F}(\mathbf{u}, \beta, H) = 0 \quad \leftarrow \quad \text{flow model}$$

How to compute **total derivatives** of the functional w.r.t. the parameters?

$\boxed{\text{Solve State System}}$ $\quad \mathcal{F}(\mathbf{u}, \beta, H) = 0$

$\boxed{\text{Solve Adjoint System}}$ $\quad \langle \mathcal{F}^*_{\mathbf{u}}(\boldsymbol{\lambda}), \, \boldsymbol{\delta}_{\mathbf{u}} \rangle = \mathcal{J}_{\mathbf{u}}(\boldsymbol{\delta}), \quad \forall \boldsymbol{\delta}_{\mathbf{u}}$

$\boxed{\text{Total derivatives}}$ $\quad \mathcal{G}(\delta_\beta, \delta_H) = \mathcal{J}_{(\beta,H)}(\delta_\beta, \delta_H) - \langle \boldsymbol{\lambda}, \mathcal{F}_{(\beta,H)}(\delta_\beta, \delta_H) \rangle$

$\boxed{\text{Derivative w.r.t. } \beta}$ $\quad \mathcal{G}_1(\delta_\beta) = \alpha_\beta \int_\Sigma \nabla \beta \cdot \nabla \delta_\beta \, ds - \int_\Sigma \delta_\beta \, \mathbf{u} \cdot \boldsymbol{\lambda} \, ds$

Estimation of ice sheet initial state

Algorithm and Software tools used

ALGORITHM	SOFTWARE TOOLS
Basal nonuniform triangular mesh	Triangle
Linear Finite Elements on tetrahedra	LifeV
Quasi-Newton optimization (L-BFGS)	Rol
Nonlinear solver (Newton method)	NOX
Krylov linear solvers	AztecOO/IfPack

Trilinos

Details:

Regularization terms: Tikhonov

L-BFGS initialized with Hessian of the regularization terms

$$\left(\frac{1}{2}\beta^T L \beta \quad \to \quad L\right)$$

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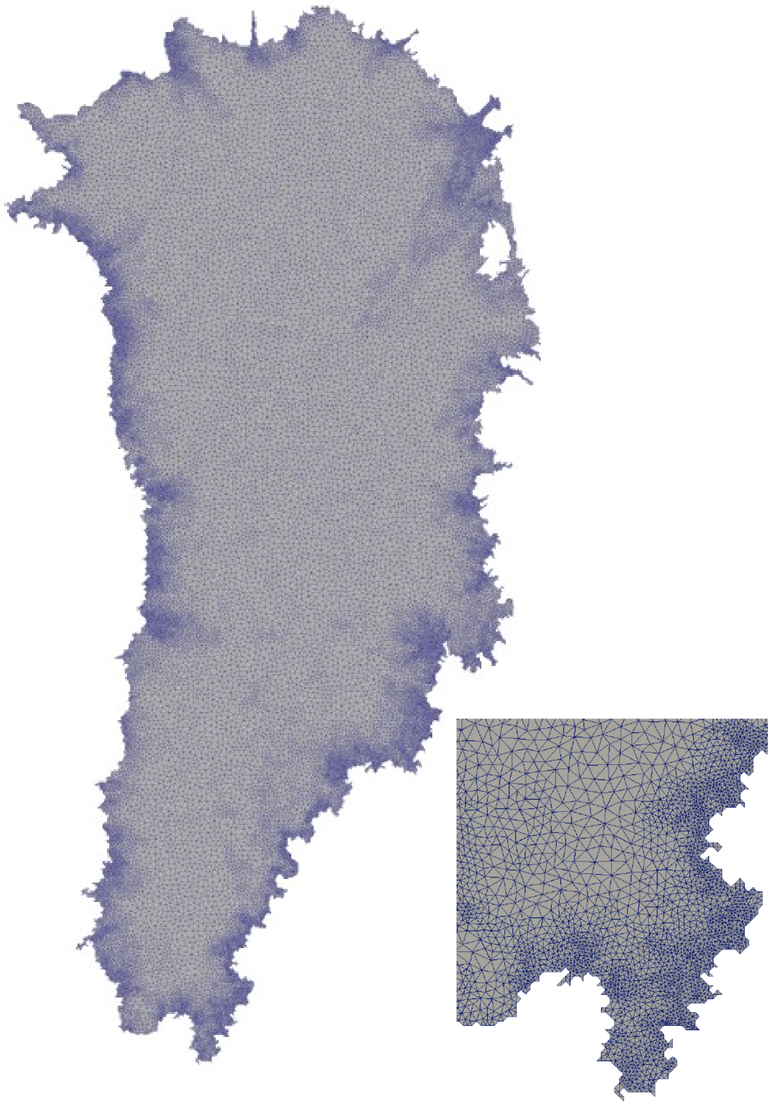
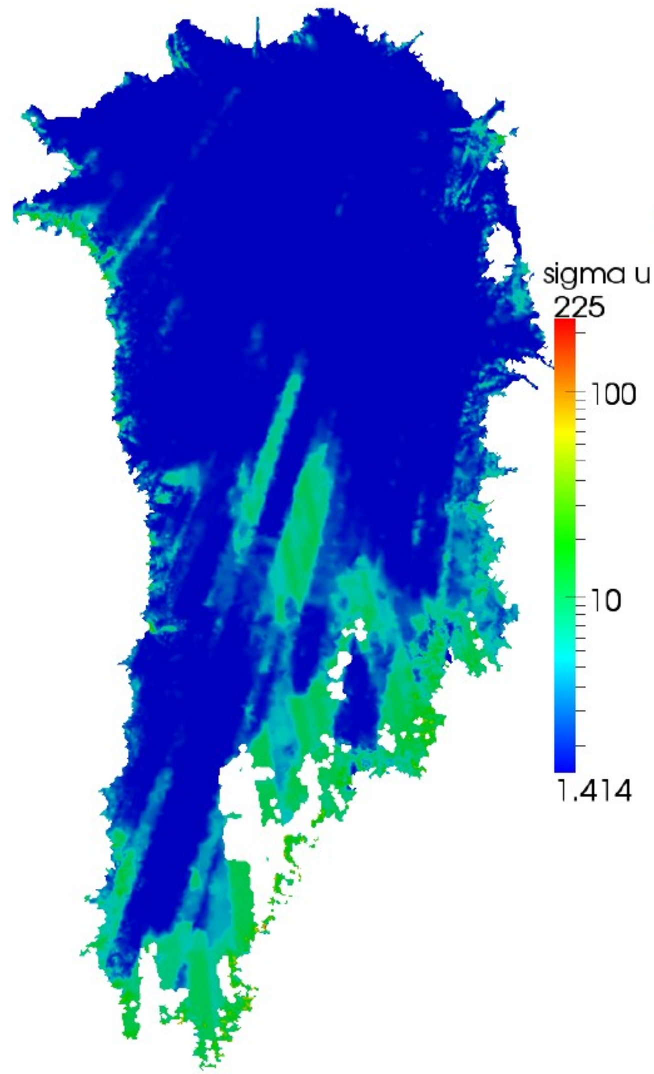
Estimation of the initial state of Greenland ice sheet

Grid and RMS of velocity and errors associated with velocity and thickness observations

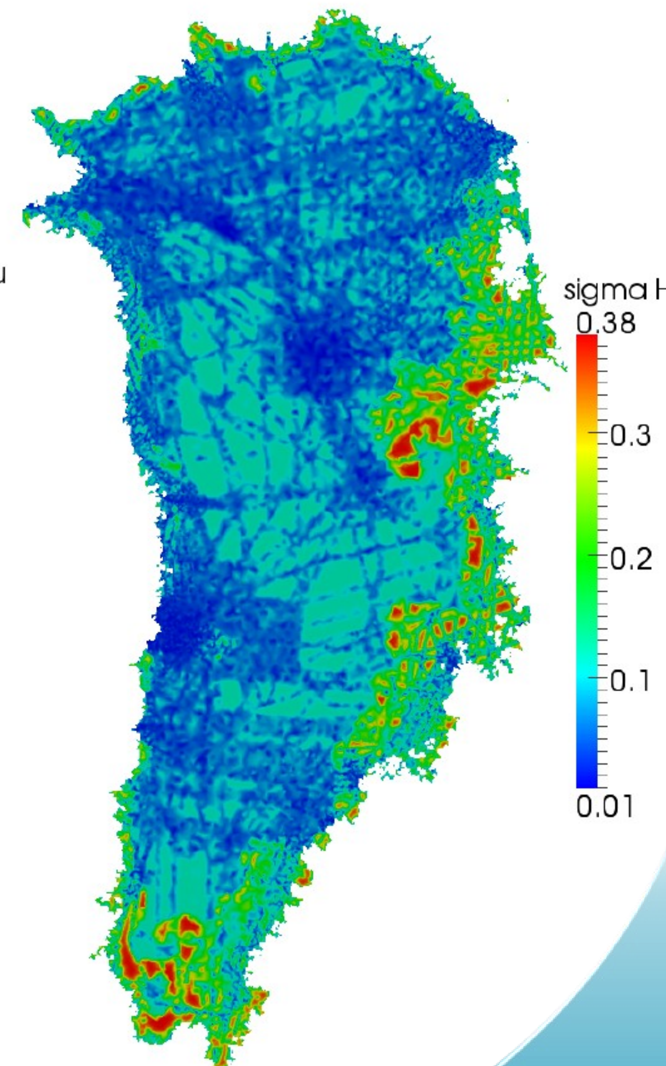
Grid



Velocity RMS (m/yr)



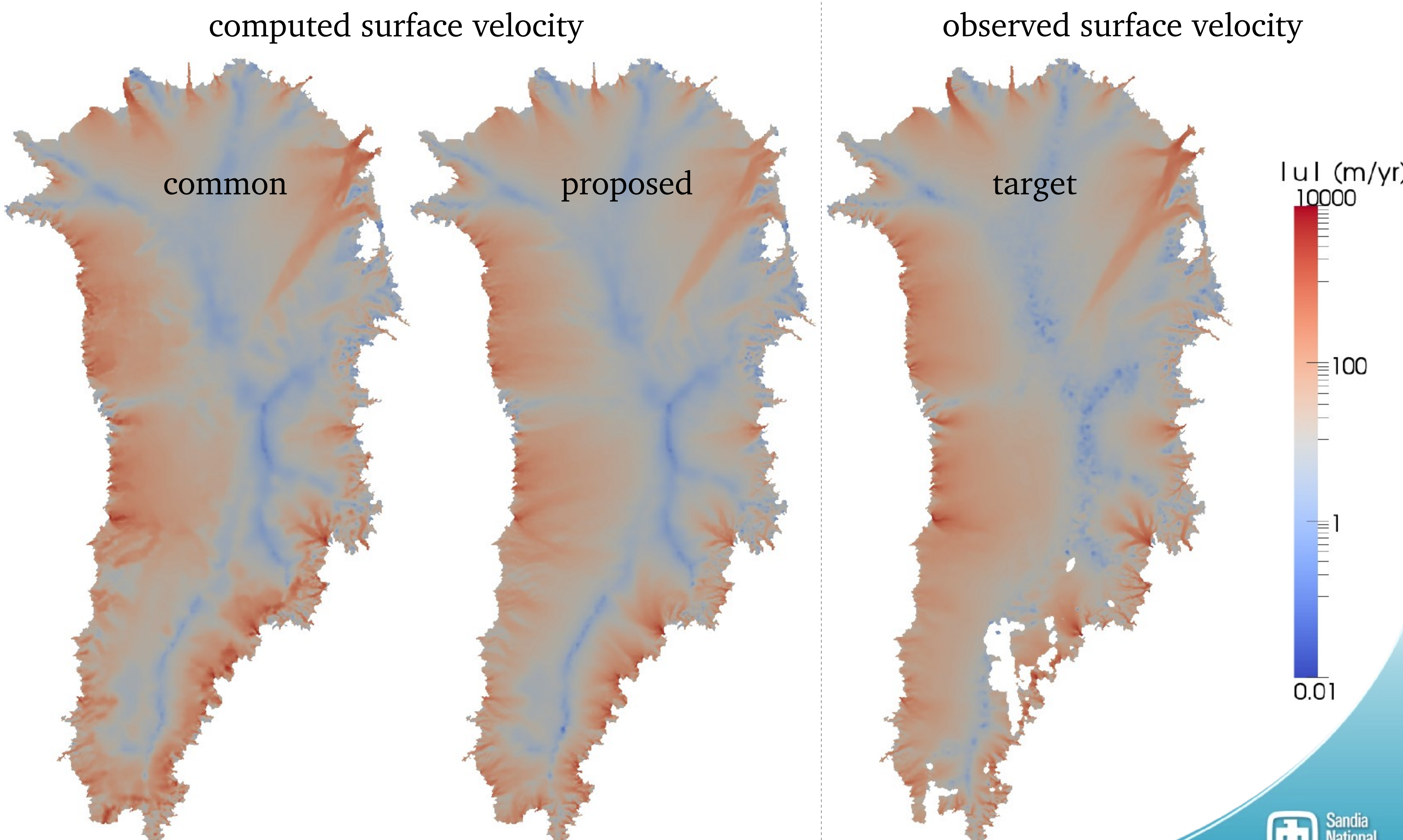
Thickness RMS (km)



Geometry and fields Bamber et al.[2013], temperature computed with CISM (Shannon et al. [2013])

Estimation of the initial state of Greenland ice sheet

Inversion results: surface velocities

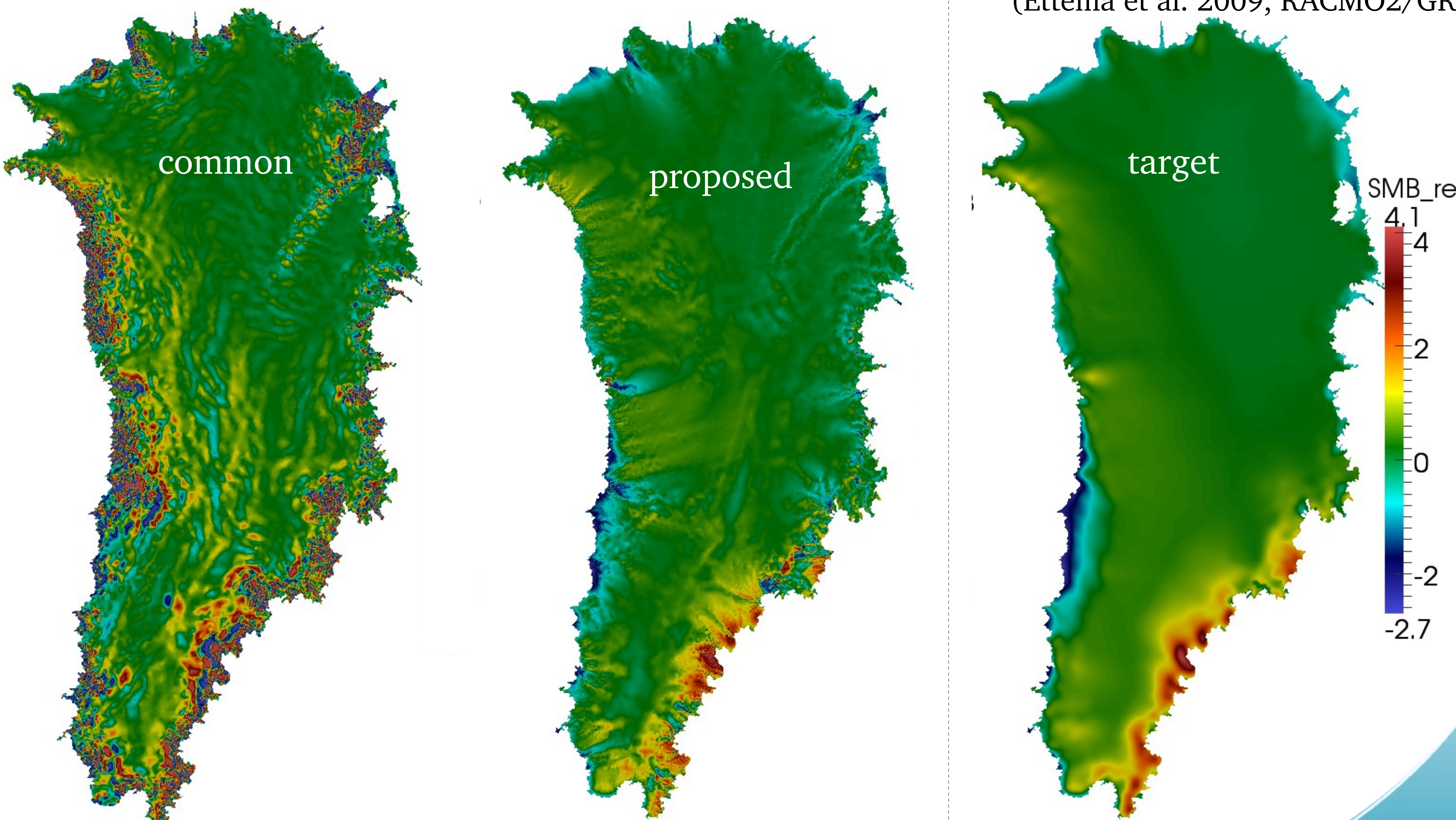


Estimation of the initial state of Greenland ice sheet

Inversion results: surface mass balance (SMB)

SMB needed for equilibrium

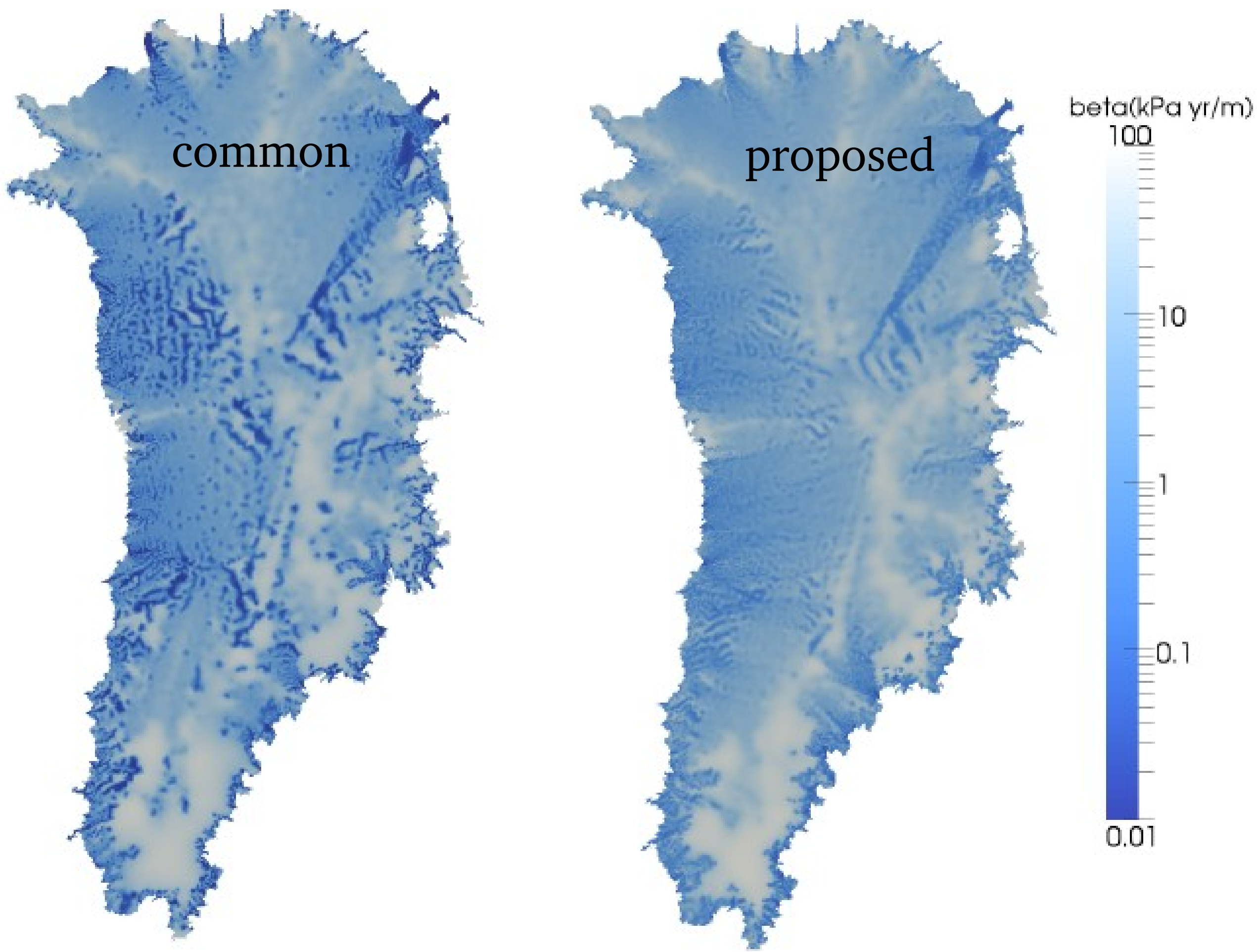
SMB from climate model
(Ettema et al. 2009, RACMO2/GR)



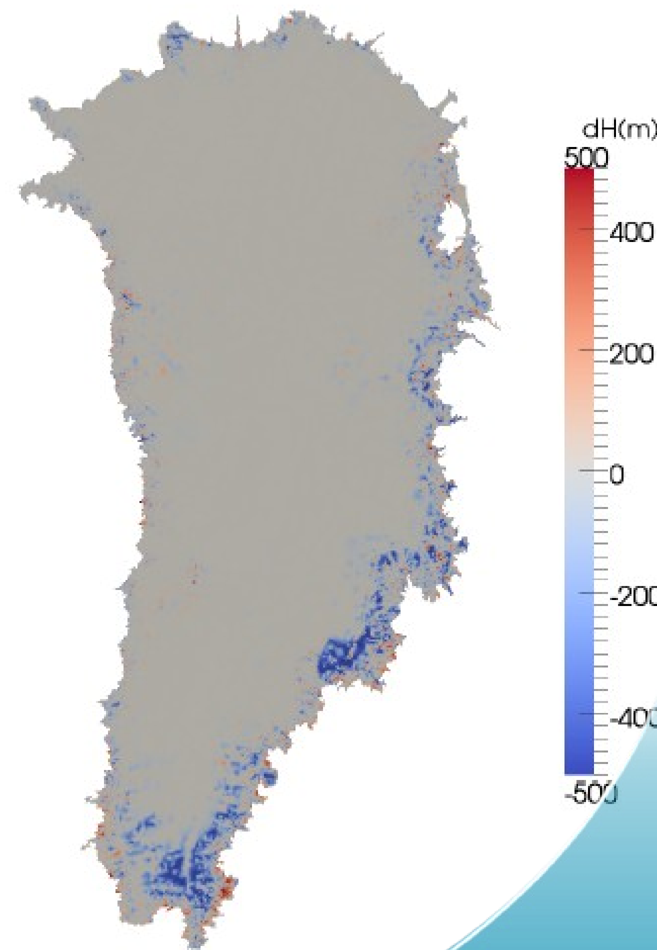
Estimation of the initial state of Greenland ice sheet

Estimated beta and change in topography

recovered basal friction



difference between recovered and observed thickness



Implementation of adjoints capability in newer code Albany-FELIX

(w/ E. Phipps, A. Salinger, D. Ridzal and D. Kouri [SNL])

Albany-Felix: Albany ice sheet solver

Why?

- to exploit Automatic Differentiation for computing derivatives
- to exploit Albany/Trilinos ecosystem (e.g. for UQ capabilities using Dakota)
- to use in-house software (better maintainability)

Features:

- automatic differentiations to compute adjoints and objective functional derivatives
- coupled with ROL (Rapid Optimization Library) package in Trilinos, to perform reduced gradient based optimization
- coupling with Dakota for UQ capabilities

TODO:

- Implement Hessian to use quasi-Newton methods
- Add shape optimization to be able to invert for bedrock topography
- Improve robustness of inversion and explore different optimization strategies

(see A. Salinger's talk on Tuesday, 2:40pm, room 258, MS225)

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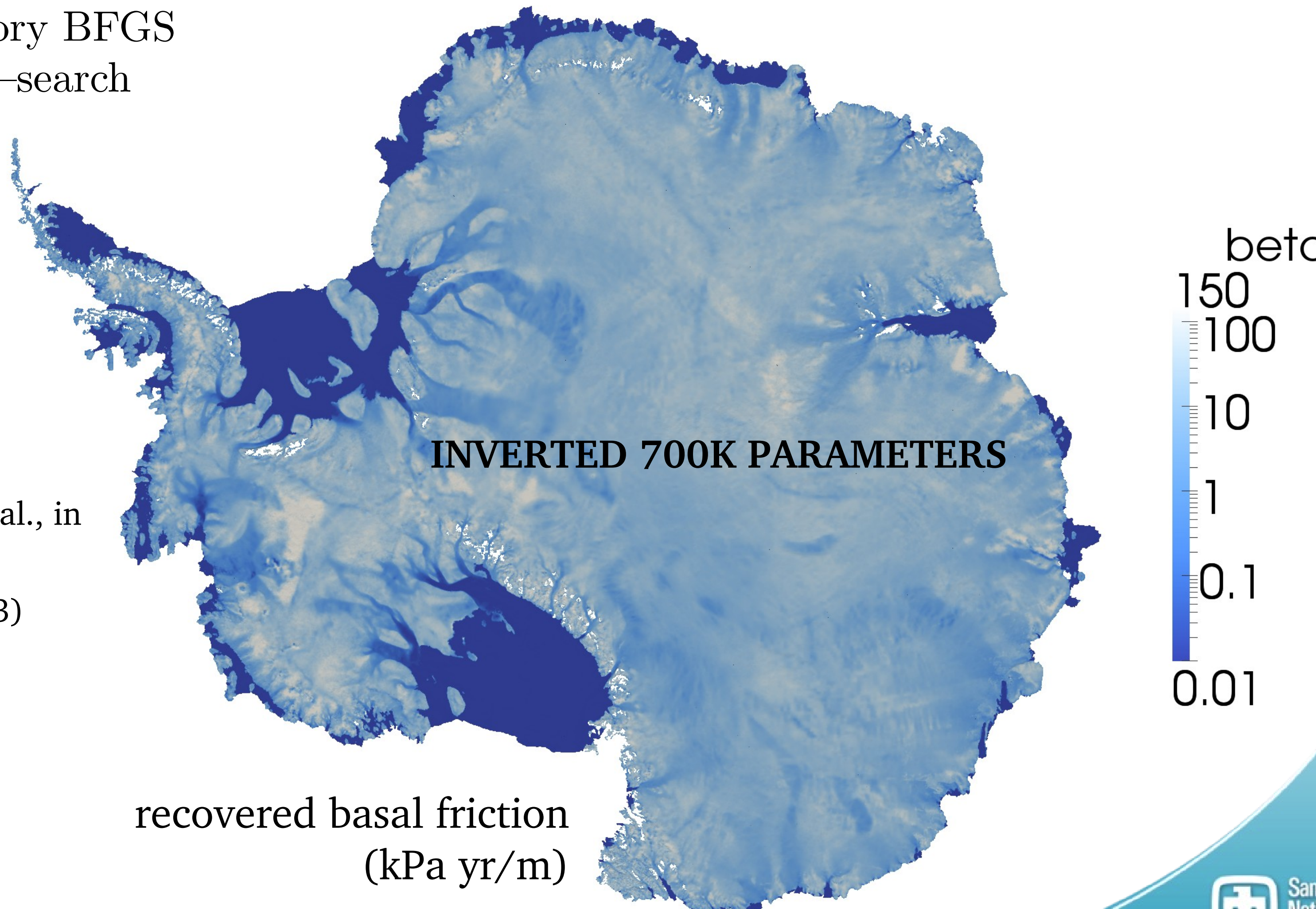
Antarctica Inversion using Albany-Piro-ROL

Objective functional: $\mathcal{J}(\mathbf{u}(\beta),\ \beta) = \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds + \ \alpha \int_\Sigma |\nabla \beta|^2 \, ds$

ROL algorithm:

- Limited–Memory BFGS
- Backtrack line–search



Gometry (Cornford, Martin et al., in prep.)

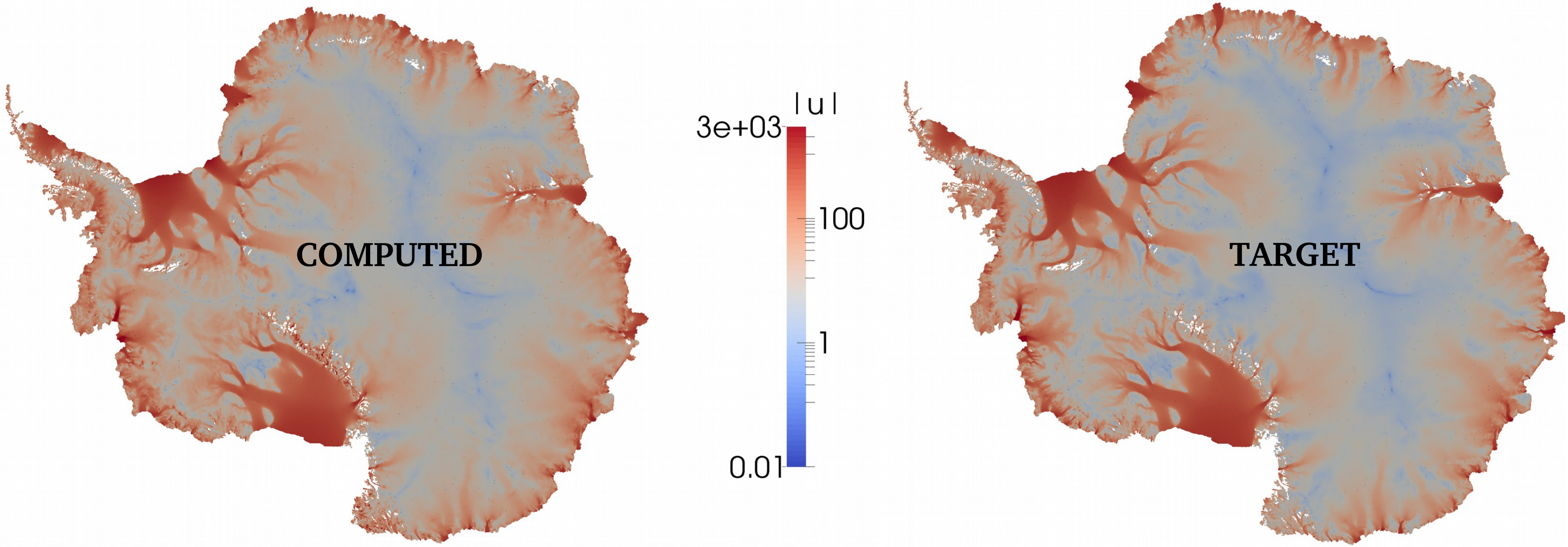
Bedmap2 (Fretwell et al., 2013)

Temperature (Pattyn, 2010)

recovered basal friction (kPa yr/m)

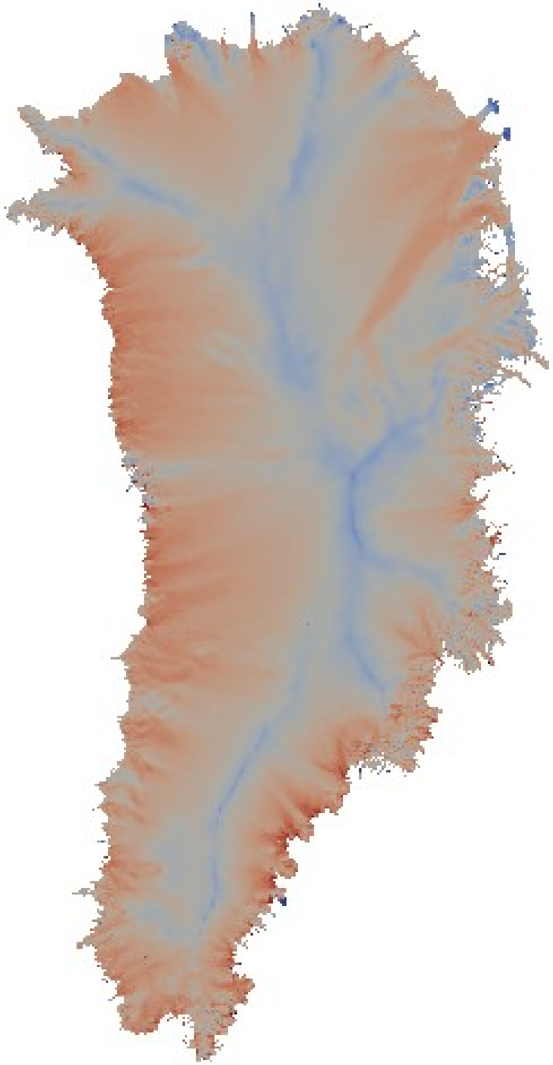
Antarctica Inversion using Albany-Piro-ROL

comparison surface velocities, computed vs. target



surface velocity magnitude (m/yr)

On-going work



- *Bayesian calibration / Uncertainty propagation (w/ M. Eldred, C. Jackson (U. Texas), J. Jakeman, I. K. Tezaur, G. Stadler (Courant) , A. Salinger)*

- *Use Hessian of deterministic inversion to estimate Covariance of basal friction distribution (N. Petra, G. Stadler, O. Ghattas)*

Thank you